Collisionless Resistivity in the Earth’s Magnetosphere

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• Resistivity and dissipation
• Ohm’s law and frozen-in condition
• Magnetic field motion (diffusion and convection)
• Magnetosphere
• Examples of breakdown
• Dissipation issue with space plasmas
• Resistivity models (Collisionless)
• Inertial resistivity
• Recap
Resistivity and Dissipation

- Flow of charge through conductor
- Electrons undergo collisions
- Material provides resistance
- Produces dissipation
Fluid picture

\[ nm \left[ \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = qnE - \nabla p - mnv u \]

\[
\begin{align*}
J & \equiv nqu \\
0 & = qnE - mnv u \\
J & = \frac{nq^2}{mv} E
\end{align*}
\]

Basic Ohm’s Law
Generalized Ohm’s Law

\[ E + u \times B = u_H \times B - \frac{1}{nq} \nabla \cdot P + \frac{m}{nq^2} \frac{\partial J}{\partial t} + \eta J \]

- LHS is the frozen-in condition
- RHS breaks the frozen-in condition
- For solar corona for example: RHS order of magnitude
  \[10^{-7}, 10^{-9}, 10^{-10}, 10^{-8}\]
- Pretty ideal (magnetic field frozen-in to the plasma)
Magnetic field motion

\[ \mathbf{E} + \mathbf{u} \times \mathbf{B} = \mathbf{u}_H \times \mathbf{B} - \frac{1}{nq} \nabla \cdot \mathbf{P} + \frac{m}{nq^2} \frac{\partial \mathbf{J}}{\partial t} + \eta \mathbf{J} \]

\[
\eta \mathbf{J} = \mathbf{E} + \mathbf{u} \times \mathbf{B}
\]

\[
\frac{\eta}{\mu_0} \nabla \times \mathbf{B} = \mathbf{E} + \mathbf{u} \times \mathbf{B}
\]

\[
-\frac{\eta}{\mu_0} \nabla^2 \mathbf{B} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B})
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})
\]

Fig. 5.1. Diffusion of magnetic field lines.

Fig. 5.2. Magnetic field lines moving with the plasma.
Anomaly: Solar Eruption
Anomaly: Magnetotail

Satellite: IMAGE
Depicts: Animated sequence of auroral activity
Copyright: IMAGE/NASA

Issues with space plasmas

• Dissipation does occur (breaking of frozen-in condition)
• Plasma is collisionless
• Collisional resistivity too low
  – Solar wind
  – Could take years for eruption
• Where does the fast dissipation come from?

Solution: Model dissipation through current sheets where anomalous resistivity is present
Current Sheet (diffusion region)

- Magnetic field (B) weak or zero (parabolic)
- Current density (J) maximum where B minimum (Ampère's circuital law)

Sources of Resistivity

• Collisions (too rare)
• Turbulence (noise)
  – Wave particle interaction
• Inertial effects in highly curved (far from dipolar) magnetic fields
Dipolar Field

- Field strength minimum at equator
- Maximum at pole (mirror)
- Particle trapping (Van Allen belts)
- Perform three motions (adiabatic invariants)
Far from dipolar fields
Inertial effects

- Ohm’s law

\[ E + u \times B = u_H \times B - \frac{1}{nq} \nabla \cdot P + \frac{m}{nq^2} \frac{\partial J}{\partial t} + \eta J \]

\[ E + u \times B = E = \frac{m}{nq^2} \frac{\partial J}{\partial t} + \eta J \]

\[ \vec{E} = \frac{m}{ne^2} \frac{\partial \vec{J}}{\partial t} + \eta J \]

where

\[ \eta = \frac{1}{\sigma} \]

\[ \sigma = \frac{ne^2}{m \tau_c} \]

\[ \frac{m}{ne^2} \frac{\partial \vec{J}}{\partial t} \approx \frac{m}{ne^2} \frac{\vec{J}}{\tau} \]

\[ \vec{J} = \sigma_c \left( 1 - \exp \left( -\frac{t}{\tau_c} \right) \right) \vec{E} \]

\[ \sigma_i(\tau) = \sigma_c \left( 1 - \exp \left( -\frac{\tau}{\tau_c} \right) \right) \]
Inertial effects

- Important when particle lifetime small compared to collision mean free time

\[
\bar{J} = \sigma_c \left(1 - \exp\left(-\frac{t}{\tau_c}\right)\right) \bar{E}
\]

\[
\sigma_i(\tau) = \sigma_c \left(1 - \exp\left(-\frac{\tau}{\tau_c}\right)\right)
\]

\[
\tau \ll \tau_c \Rightarrow \sigma_i \approx \sigma_c \left(1 - 1 + \frac{\tau}{\tau_c}\right) \Rightarrow \sigma_i \approx \frac{\sigma_c \tau}{\tau_c}
\]

\[
\tau \gg \tau_c \Rightarrow \sigma_i \approx \sigma_c
\]

\[
\tau \to 0 \Rightarrow \sigma_i = 0
\]

- Inertial conductivity is orders of magnitude smaller than collisional counterpart
- Means for sufficiently fast dissipation
Particle trajectory in Y-line
Recap

- Dissipation - an important mechanism in space plasmas
- Plasma being collisionless poses a problem
- Noise (particle scattering by waves) and particle inertia can be considered in current sheets
- Produces dissipation rate that can explain bursty events